Section 5.4

Fundamental Theorem of Calculus

The Fundamental Theorem of Calculus (PART 1)

Evaluating Definite Integrals

If f is continous on [a,b], then the function

$$F(x) = \int_a^b f(t)\,dt$$

has a derivative at every point x in [a,b] and

$$\frac{dF}{dx} = \frac{d}{dt}\int_a^x f(t)\,dt = f(x)$$

Find

$$\frac{d}{dx}\int_{-\pi}^{x} \cos t \, dt \qquad \text{and} \qquad \frac{d}{dx}\int_{0}^{x} \frac{1}{1+t^2}\, dt$$

More Examples
Find g'(x)

a.) $g(x) = \int_1^x (t^2 + 1)^3 \, dt$

b.) $g(x) = \int_x^2 \cos(t^2 + t) \, dt$

Find dy/dx

$$y = \int_{1}^{x^2} \cos(t)\, dt$$

The Fundamental Theorem with the Chain Rule

Find g'(x)

$$g(x) = \int_1^{\sqrt{x}} \frac{a^2}{a^2 + 1}\, da$$

Find dy/dx

a.) $y = \int_{x}^{5} 3t \sin t \, dt$

b.) $y = \int_{2x}^{x^2} \frac{1}{2 + e^t} \, dt$

$$\frac{d}{dx}\int_x^7 \sqrt{\sin t}\; dt$$

$$\frac{d}{dx}\int_{x}^{e^x}\frac{t^2+1}{2t}\,dt$$

$$\frac{d}{dx}\int_{4}^{x}\sqrt{\sin t^{3} - 4t}\;dt$$

$$\frac{d}{dx}\int_{2}^{\cos x} \sqrt{t}\, dt$$

$$\frac{d}{dx}\int_{3x}^{5} t^3 + 1 \, dt =$$

$$\frac{d}{dx}\int_{x}^{2x^2} \sin t \, dt$$

Find f'(x)

$$f(x) = \int_{2}^{x} \sin t^2 \, dt$$

Find the derivative

$$g(x) = \int_{\tan x}^{x^2} \frac{1}{\sqrt{2 + t^4}}\, dt$$

Find a function $y = f(x)$ with a derivative

$$\frac{dy}{dx} = \tan x$$

that satisfies the condition $f(3) = 5$

Find a function f(x) that has a derivative of

$$\frac{\sin x}{\sqrt{x}}$$

and passes through the point (7,-2)

Find a function f(x) that has a derivative

$$\sqrt{\cos x}$$

and passes through the point $(-\pi, 4)$

Find a function with a derivative of $2x^2 + 4x$ that passes through (2,9)

Evaluate $$\int_{-1}^{3} (x^3 + 1)\, dx$$

Fundamental Theorem of Calculus (PART 2)

If f is continuous at every point of [a,b], and if F is any antiderivative of f on [a,b], then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

Also called the INTEGRAL EVALUATION THEOREM

SUBSTITUTE!!!

Find the area of the region between the curve $y = 4 - x^2$, $0 \leqslant x \leqslant 3$, and the x axis

How to Find the Total Area Analytically

To find the area between the graph of $y = f(x)$ and the x - axis over the interval $[a,b]$ analytically,

1.) Partition $[a,b]$ with the zeros of f

2.) Integrate f over each subinterval

3.) Add the absolute values of the integrals

Find the area of the region between the curve $y = x \cos 2x$ and the x-axis over the interval $-3 \leq x \leq 3$.

Antiderivatives...

$3x^2$

$2x^3$

$\cos x$

$\sec^2(2x)$

e^x

$$\sqrt{x}$$

$$\frac{1}{x + 1}$$

$$\frac{-1}{\sqrt{1 - x^2}}$$

$$\frac{2}{x^3}$$

Find the total area under the graph of $y = -x^2 + 4$ between $x = 0$ and $x = 5$

$$\left|\int_0^2 -x^2+4\,dx\right| = \left|-\frac{x^3}{3}+4x\Big|_0^2\right| = \left|\frac{-2^3}{3}+8\right| = \frac{16}{3}$$

$$\left|\int_2^5 -x^2+4\,dx\right| = \left|-\frac{x^3}{3}+4x\Big|_2^5\right| = \left|\left(\frac{-5^3}{3}+20\right)-\left(\frac{-2^3}{3}+8\right)\right|$$

$$= \frac{97}{3}$$

Evaluate $\displaystyle\int_1^5 2x^2 + x - 2\,dx$

$$\frac{2x^3}{3} + \frac{x^2}{2} - 2x \Big|_1^5 = \left(\frac{2(5)^3}{3} + \frac{5^2}{2} - 2(5)\right) - \left(\frac{2(1)^3}{3} + \frac{1^2}{2} - 2(1)\right)$$

$$= \left(\frac{2(125)}{3} + \frac{25}{2} - 10\right) - \left(\frac{2}{3} + \frac{1}{2} - \right.$$

$$= \left(\frac{250}{3} + \frac{25}{2} - 10\right) - \left(\frac{2}{3} + \frac{1}{2} - \right.$$

$$= \frac{260}{3}$$

Evaluate

$$\int_{-\pi}^{\frac{\pi}{2}} \cos x \, dx = 1$$

Evaluate

$$\int_0^4 2e^x\,dx = 2e^x\Big|_0^4$$

$$= 2e^4 - 2e^0$$

$$= 2e^4 - 2$$

Evaluate $\displaystyle\int_{2}^{x} t^2 + 4t - 3\, dt$

$$= \frac{t^3}{3} + \frac{4t^2}{2} - 3t \Big|_2^x$$

$$= \frac{t^3}{3} + 2t^2 - 3t \Big|_2^x = \left(\frac{x^3}{3} + 2x^2 - 3x\right) - \left(\frac{8}{3} + 8 - 6\right)$$

$$= \frac{x^3}{3} + 2x^2 - 3x - \frac{14}{3}$$

$$\frac{d}{dx}\ x^3 + 2x^2 - 3x - \frac{14}{3}$$

$$\int_0^{x^2} \cos t \, dt =$$

If $$\int_0^x f(t)\,dt = x\cos(\pi x)$$

Find f(4)

Suppose $\displaystyle\int_1^x g(t)\,dt = 2x^2 - 3x + 1$

Find $g'(x)$

If $f(x) = \int_{a}^{x} e^{t^2}\, dt$

If $f(0) = 5$. Find $f(3)$

$$F(x) = \int_{2}^{x} t^2 + 4t \, dt$$

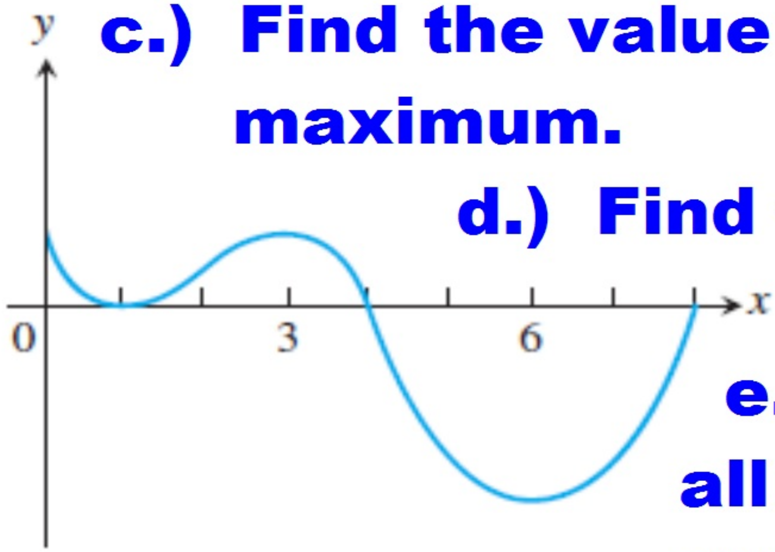
Find the equation of the tangent to F(x) at x = 5

The graph of a continuous function f with domain [0,8] is shown below. Let h be the function defined by $h(x) = \displaystyle\int_1^x f(t)\,dt$

a.) Find h(1)

b.) Is h(0) positive or negative?

c.) Find the value of x for which h(x) is a maximum.



d.) Find the value of x for which h(x) is a minimum.

e.) Find the x-coordinates of all points of inflections of the graph of y = h(x).

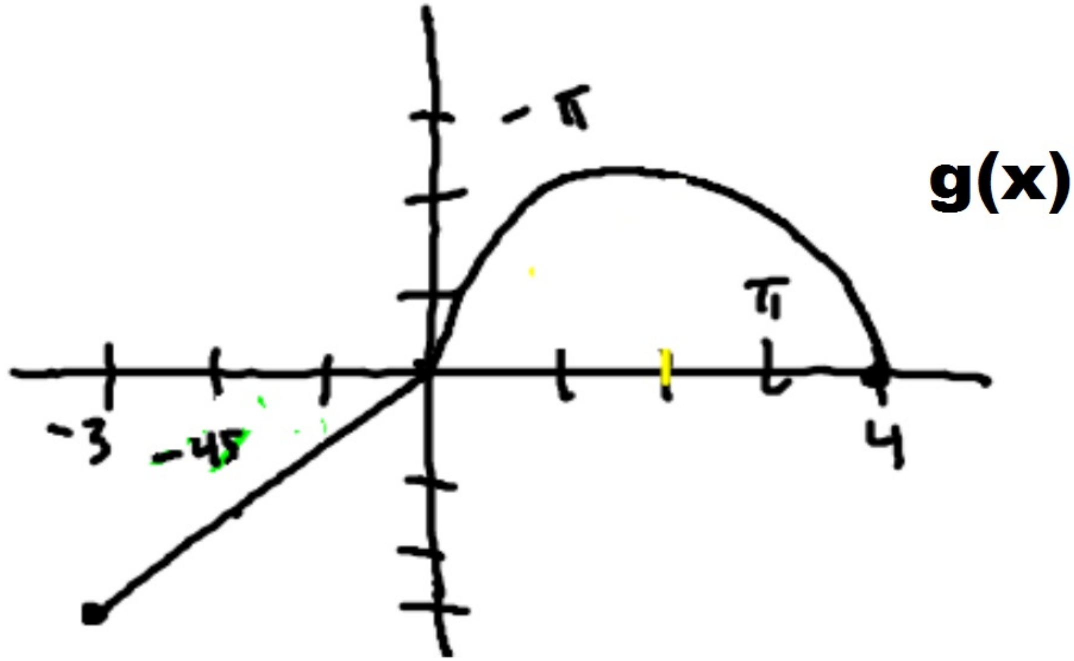
$$f(x) = \int_{2}^{x} \sin t^2 \, dt \qquad [-\pi,\pi]$$

When is f(x) increasing?

Maximum?

Is f(x) concave up or concave down?

POI?



1.) Find H(0) =

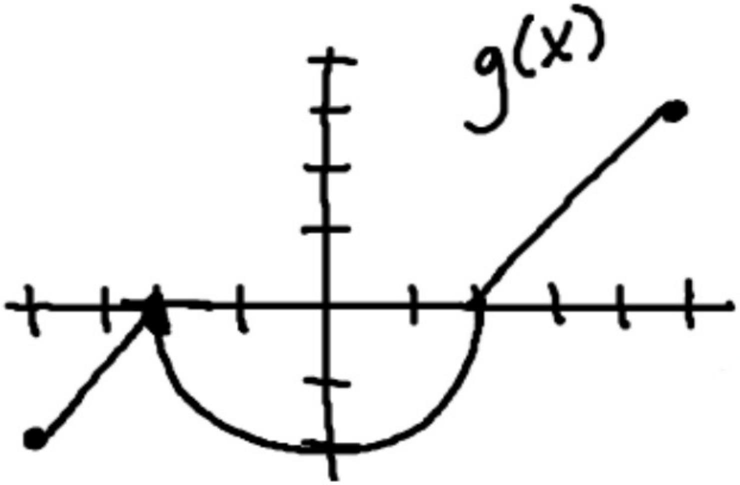
2.) When is H(x) increasing?

3.) When does H(x) have a minimum?

4.) Is x = -3 or x = 4 the absolute max?

5.) Find H" (-2)

6.) H'(-1) =



1.) Find H(2), H(0), H(5)

2.) Where is H(x) increasin concave down?

3.) Where does H(x) have an absolute maximum?

Let $H(x) = \int_2^x g(t)\,dt$

4.) Where does H(x) have point(s) of inflection?

5.) Find H'(0)

6.) Find H''(3)

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